New type of closed timelike curves in quantum gravity

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CONTENTS

QUANTUM SWITCH PROTOCOL

Introduced by the quantum information community [Chiribella etal, 2013], it describes a hypothetical quantum protocol "without a definite causal structure":

- an example of a higher order quantum operation,
- Alice and Bob perform operations U and V on a particle in initial state $|P\rangle$,
- the order of these operations is coupled to another quantum system (the Control), which is a qubit, so that

$$U_{\text{switch}} = |0\rangle_C\langle 0| \otimes UV + |1\rangle_C\langle 1| \otimes VU,$$

The concept of a "higher order quantum operation" comes from category theory, motivating the idea that one should have full quantum control on only over *physical systems*, but also over the *operations* on physical systems:

$$\begin{tikzcd}
|\Psi_1\rangle \arrow[r, bend left, "U"] \arrow[r, bend right, "V"'] & |\Psi_2\rangle \arrow[r, bend left, "V"] \arrow[r, bend right, "U"'] & |\Psi_3\rangle
\end{tikzcd}$$

QUANTUM SWITCH PROTOCOL

In principle, one can use $U_{\mathbf{switch}}$ to arrange a quantum superposition of the order of operations A and B:

- start from the initial total state $|\Psi_i\rangle = |C\rangle \otimes |P\rangle$, with $|C\rangle = \frac{1}{\sqrt{2}}\,(|0\rangle + |1\rangle)$,
- apply the switch operation to obtain the final state

$$|\Psi_f\rangle = \frac{1}{\sqrt{2}}|0\rangle \otimes UV|P\rangle + \frac{1}{\sqrt{2}}|1\rangle \otimes VU|P\rangle\,,$$

- project the control in the superposed basis $|\pm\rangle = \frac{1}{\sqrt{2}}\,(|0\rangle \pm |1\rangle)$,
- depending on the outcome for the control being $+$ or $-$, the particle will end up in the state

$$\frac{1}{2}\,(UV + VU)\,|P\rangle \qquad \text{or} \qquad \frac{1}{2}\,(UV - VU)\,|P\rangle\,,$$

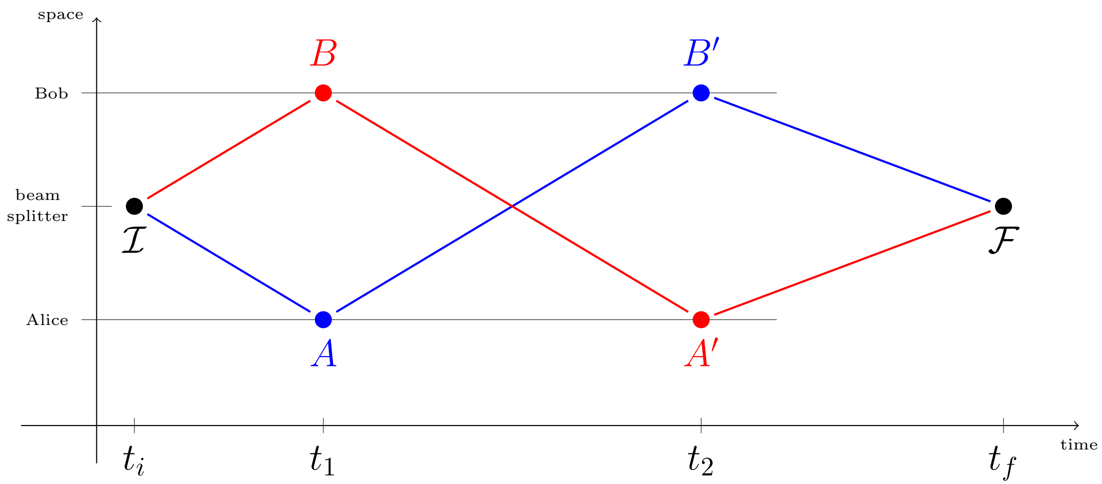
- both resulting states represent a quantum superposition of the order of operations U and V.

OPTICAL SWITCH

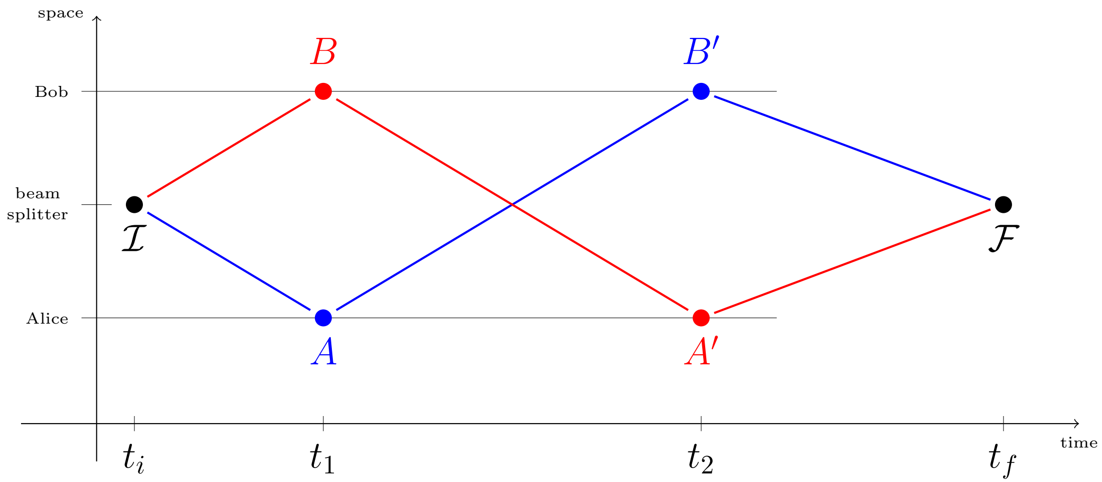
Can one implement this in experiment? Under certain additional assumptions, sure!

- experiments in quantum optics [Procopio etal, 2015; Rubino etal, 2017; Rubino etal, 2022],
- particle $\rightarrow$ a polarized photon,
- the control $\rightarrow$ photon trajectory,
- operations U and V $\rightarrow$ rotations of the polarization.

Such experimental realization is called the "optical switch". A spacetime diagram of the process:



OPTICAL SWITCH



Controversy about the "order of operations" vs. "causal order":

- blue history: $\mathcal{I} \prec_{\mathcal{C}} A \prec_{\mathcal{C}} B' \prec_{\mathcal{C}} \mathcal{F}$,
- red history: $\mathcal{I} \prec_{\mathcal{C}} B \prec_{\mathcal{C}} A' \prec_{\mathcal{C}} \mathcal{F}$.

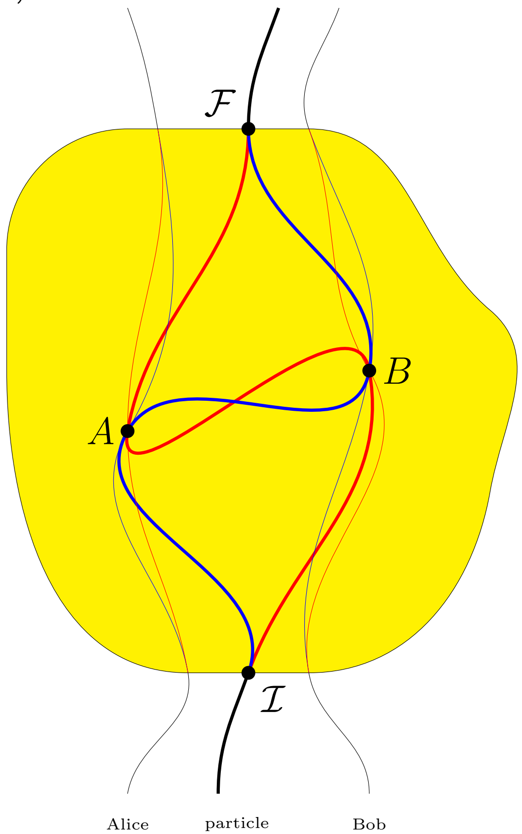
Can we have both $A \equiv A'$ and $B \equiv B'$ or not?

- If yes $\rightarrow$ quantum superposition of causal order!!
- If not $\rightarrow$ quantum superposition of order of operations (but not causal order).

GRAVITATIONAL SWITCH

Can one obtain a genuine superposition of causal orders, i.e., no definite causal structure? Under certain additional assumptions, sure!

- Optical switch →
  "simulation" but not "implementation".
- Gravitational field →
  local light cone structure.
  Superpositions of gravitational fields →
  superpositions of causalities!
- Proper implementation →
  the gravitational switch!!
  [Zych etal, 2019; Paunković, MV, 2020].



CTC CONSTRUCTION

The gravitational switch can be applied to implement closed timelike curves, in a novel way:

$$|G_I\rangle|\Psi_I\rangle \xrightarrow{U_1} \alpha_I|G_B\rangle|\Psi_I\rangle + \beta_I|G_R\rangle|\Psi_I\rangle\,.$$

$$\alpha_I|G_B\rangle|\Psi_I\rangle + \lambda_B\beta_B|G''_R\rangle V|\Psi_B\rangle \xrightarrow{U_2} \lambda_A|G_A\rangle|\Psi_A\rangle\,.$$

$$\lambda_A|G_A\rangle|\Psi_A\rangle \xrightarrow{U_3} \lambda_A|G_A\rangle U|\Psi_A\rangle\,,$$

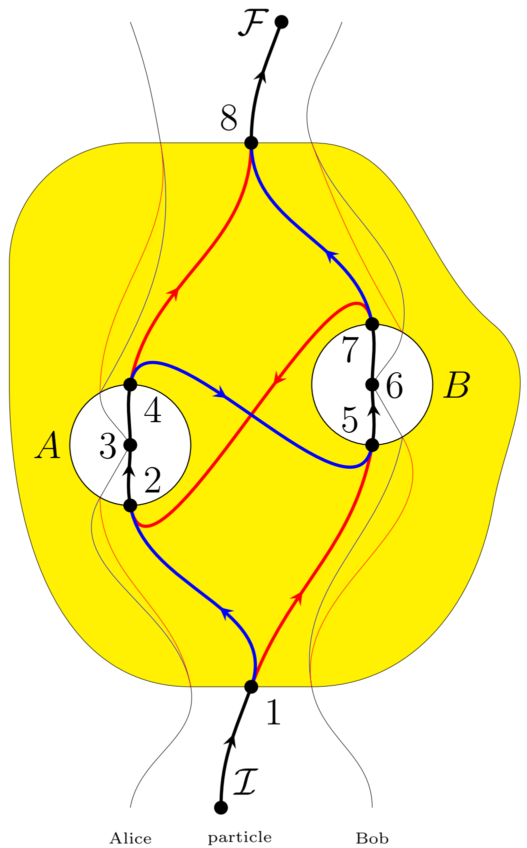
$$\lambda_A|G_A\rangle U|\Psi_A\rangle \xrightarrow{U_4} \lambda_A\Big(\alpha_A|G'_B\rangle + \beta_A|G'_R\rangle\Big)U|\Psi_A\rangle\,.$$

$$\beta_I|G_R\rangle|\Psi_I\rangle + \lambda_A\alpha_A|G'_B\rangle U|\Psi_A\rangle \xrightarrow{U_5} \lambda_B|G_B\rangle|\Psi_B\rangle.$$

$$\lambda_B|G_B\rangle|\Psi_B\rangle \xrightarrow{U_6} \lambda_B|G_B\rangle V|\Psi_B\rangle\,,$$

$$\lambda_B|G_B\rangle V|\Psi_B\rangle \xrightarrow{U_7} \lambda_B\Big(\alpha_B|G''_B\rangle + \beta_B|G''_R\rangle\Big)V|\Psi_B\rangle\,,$$

$$\lambda_A\beta_A|G'_R\rangle U|\Psi_A\rangle + \lambda_B\alpha_B|G''_B\rangle V|\Psi_B\rangle \xrightarrow{U_8} |G_O\rangle|\Psi_O\rangle.$$



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A so-called *trivial solution* exists:

$$U = V = I \,,$$

$$|\Psi_A\rangle = |\Psi_B\rangle = |\Psi_O\rangle = |\Psi_I\rangle \,,$$

$$U_4 = U_2^{-1} \,, \qquad U_7 = U_5^{-1} \,, \qquad U_8 = U_1^{-1} \,,$$

$$|G'_R\rangle = |G''_R\rangle = |G_R\rangle \,, \quad |G'_B\rangle = |G''_B\rangle = |G_B\rangle \,,$$

$$\begin{aligned} |G_B\rangle &= \alpha^{-1}U_1|G_I\rangle - \alpha^{-1}\beta|G_R\rangle \,, \\ |G_A\rangle &= U_2U_1|G_I\rangle \,, \\ |G_B\rangle &= U_5U_1|G_I\rangle \,, \\ |G_O\rangle &= |G_I\rangle \,, \end{aligned}$$

$$|\lambda_A| = |\lambda_B| = 1 \,, \qquad \beta = \sqrt{1-\alpha^2} \,,$$

$|G_I\rangle$, $|G_R\rangle$, α, U_1, U_2, U_5, $|\Psi_I\rangle$ are arbitrary.

The trivial solution is important since it demonstrates the self-consistency of the system, and that the set of solutions is nonempty.

CTC CONSTRUCTION

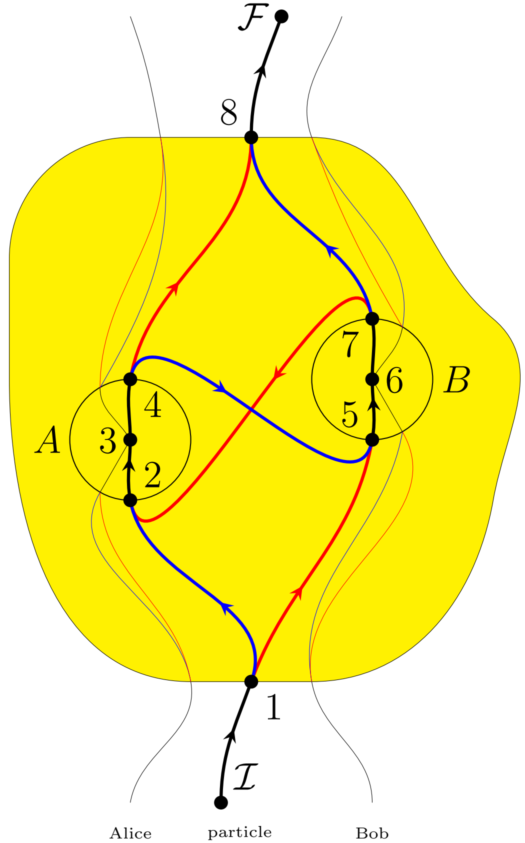
Even more general configurations of the gravitational switch have CTC solutions, including nontrivial ones:

- Three classes of nontrivial solutions found, depending on several parameters.
- General structure — one part of the solution describes the particle entering and exiting the yellow blob, and the other part describes the particle cycling within a closed loop

$$2 \to 3 \to 4 \to 5 \to 6 \to 7 \to 2\,.$$

- Rough structure of the implicit solution:

$$\Big(1 + c_1 UV\Big)|\Psi\rangle = \Big[c_2 U + c_3 UV + c_4 UVU\Big]|\Psi_I\rangle\,,$$
$$c_1, c_2, c_3, c_4 \in \mathbb{C}\,.$$



CONCLUSIONS

- **Quantum information community came up with an interesting toy example process — the quantum switch.**
- **The optical version of the switch is not an implementation but a simulation.**
- **The proper implementation of the switch can be done in the context of quantum gravity — the gravitational switch.**
- **The ideas behind the gravitational switch can be modified and utilized to construct various CTC solutions.**
- **The CTC solutions do not require exotic topology, but exist in ordinary globally hyperbolic spacetimes.**
- **Arguably, it is easier to put gravity into a quantum superposition than to change spacetime topology...**

Research is ongoing!

THANK YOU!